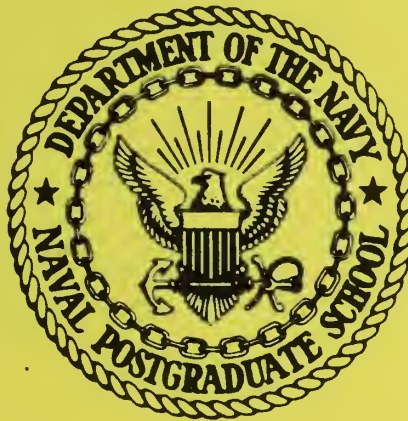


1427  
PROCESSING

NPS55-83-025

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## Monterey, California



A MULTIPLE LEG TMA PROCEDURE WITH PROGRAMS  
FOR THE HEWLETT-PACKARD HP-41CV,  
THE HEWLETT-PACKARD HP-75C,  
THE SHARP PC-1500 (TRS-80 PC-2) AND  
THE RADIO SHACK TRS-80 MODEL 100  
PORTABLE COMPUTERS

by

Rex H. Shudde

September 1983

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NAVAL POSTGRADUATE SCHOOL  
Monterey, California

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The programs in this report  
are for use within the Navy,  
and they are presented with-  
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of any kind.





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## ABSTRACT

This report contains user instructions and program listings for multiple leg Kalman Filter target motion analysis (TMA) procedure for use on the Hewlett-Packard HP-41CV, the Hewlett-Packard HP-75C, the Sharp PC-1500 (Radio Shack TRS-80 PC-2) and the Radio Shack Model 100 portable computers.



## I. INTRODUCTION

This report contains a multiple leg target motion analysis (TMA) program for each of the following portable computers: the Hewlett-Packard HP-41CV, the Hewlett-Packard HP-75C, the Sharp PC-1500 (Radio Shack TRS-80 PC-2) and the Radio Shack TRS-80 Model 100. These programs are part of a set that has been developed for portable computer evaluation.

These programs provide a means of implementing a TMA procedure that is described in Appendix I. The procedure requires at least four bearing observations. It is assumed that during an encounter, the target course and speed remain constant, and that signal transit times are zero.

This algorithm is a Kalman Filter adaption of the least squares TMA algorithm given by Forrest [Ref. 1].

## II. THE HEWLETT-PACKARD HP-41CV.

### HP-41CV USER INSTRUCTIONS.

There are four user options. These options - called NEW, LEG, FIX and AOP - are used as global labels in the HP-41CV and are assigned to the keys LN,  $\Sigma+$ , 1/x and  $\sqrt{x}$ , respectively, for access in the USER mode. Their use is given below. (Memory: SIZE = 40, 706 bytes are required).

DISPLAY	INSTRUCTION	PRESS
1.	Select USER mode.	USER
2.	<b>Start a new problem.</b>	LN (NEW)
3. BRG ERR?	Key in bearing error.	R/S
4. OWN CUS?	Key in own course.	R/S
5. OWN SPD?	Key in own speed.	R/S
6. TGT BRG?	Key in target bearing.	R/S
7. TIME?	Key in time (HH.MM).	R/S
8.	Repeat Steps 5 and 6 for each new observation.	
9.	<b>Input a new leg.</b>	$\Sigma+$ (LEG)
10. LEG BRG?	Key in new leg bearing.	R/S
11. LEG DST?	Key in distance on leg.	R/S
12.	Go to Step 4.	
13.	<b>Compute a fix.</b>	1/x (FIX)
14.	TGT CUS: (degrees)	R/S
15.	TGT SPD: (knots)	R/S
16.	TGT BRG: (degrees)	R/S
17.	TGT RNG: (meters)	R/S
18.	Next Option?	
19.	<b>Compute an AOP</b>	$\sqrt{x}$ (AOP)
20.	SIG 1: (n. mi.)	R/S
21.	SIG 2: (n. mi.)	R/S
22.	ANG: (degrees)	R/S
23.	Next Option?	

HP-41CV SAMPLE PROBLEM

The sensor on your ship has a bearing error of one degree. Your course is 160 deg. at 6 kts. At 1200 hours your sensor detects a target at 350.5 degrees. At 1204 the sensor indicates the target to be at 1.8 deg. To determine a fix, your ship makes a course change during which the course-made-good is 130 deg. and the distance-made-good is 556 meters. Your new course is 080 deg. at 6 kts. The maneuver ends at 1207, at which time the target bearing is 8.3 deg. One more observation places the target at 18.4 deg. at 1211. Estimate the target course, speed, bearing, range, and AOP at 1211 hours.

DISPLAY CONTENTS	USER RESPONSE	COMMENTS
	LN (NEW)	Start a new problem.
BRG ERR?	1 [R/S]	1 deg.
OWN CUS?	160 [R/S]	160 deg.
OWN SPD?	6 [R/S]	6 kts.
TGT BRG?	350.5 [R/S]	350.5 deg.
TIME?	12.00 [R/S]	1200 hours.
TGT BRG?	1.8 [R/S]	1.8 deg.
TIME?	12.04 [R/S]	1204 hours.
	Σ+ (LEG)	Input a new leg.
TGT BRG?		
LEG BRG?	130 [R/S]	130 deg.
LEG DST?	556 [R/S]	556 meters.
OWN CUS?	080 [R/S]	080 deg.
OWN SPD?	6 [R/S]	6 kts.
TGT BRG?	8.3 [R/S]	8.3 deg.
TIME?	12.07 [R/S]	1207 hours.
TGT BRG?	18.4 [R/S]	18.4 deg.
TIME?	12.11 [R/S]	1211 hours.
	1/x (FIX)	Compute a fix.
TGT BRG?		
TGT CUS: 123.7	[R/S]	course = 123.7 deg.
TGT SPD: 12.6	[R/S]	speed = 12.6 kts.
TGT BRG: 18.4	[R/S]	bearing = 18.4 deg.
TGT RNG: 4,023.0	[R/S]	range = 4023.0 meters.
	√x (AOP)	Compute an AOP.
NEXT OPTION?		
SIG 1: 10.40	[R/S]	sigma 1 = 10.40 n. mi.
SIG 2: 0.06	[R/S]	sigma 2 = 0.06 n. mi.
ANG: 18.39	[R/S]	angle = 18.39 deg.
NEXT OPTION?		Quit.

# HP-41CV PROGRAM LISTING

01*LBL "TMA"	47 P-R
02*LBL 99	48 STO 10
03 TONE 6	49 X<>Y
04 "NEXT OPTION?"	50 STO 11
05 RVIEW	51*LBL 01
06 GTO 99	52 RCL 31
07*LBL "BERR"	53 STO 30
08 "BRG ERR?"	54 "TIME?"
09 PROMPT	55 PROMPT
10 D-R	56 HR
11 X↑2	57 STO 31
12 STO 23	58 FS?C 10
13 RTN	59 STO 30
14 GTO 99	60 RCL 30
15*LBL "NEW"	61 -
16 SF 10	62 STO 14
17 CLRG	63 FS?C 09
18 1 E3	64 GTO 00
19 STO 00	65 RCL 26
20 STO 01	66 RCL 27
21 STO 02	67 RCL 14
22 STO 03	68 *
23 1852	69 P-R
24 STO 39	70 ST+ 28
25 XEQ "BERR"	71 X<>Y
26*LBL "CS"	72 ST+ 29
27 "OWN CUS?"	73*LBL 00
28 PROMPT	74 RCL 12
29 STO 26	75 RCL 14
30 "OWN SPD?"	76 *
31 PROMPT	77 ST+ 10
32 STO 27	78 RCL 13
33*LBL "TB"	79 RCL 14
34 "TGT BRG?"	80 *
35 PROMPT	81 ST+ 11
36 STO 32	82 RCL 14
37 1	83 ENTER↑
38 P-R	84 ENTER↑
39 STO 25	85 ENTER↑
40 X<>Y	86 RCL 02
41 STO 24	87 *
42 FC? 10	88 RCL 05
43 GTO 01	89 ENTER↑
44 32	90 +
45 RCL 32	91 +
46 X<>Y	92 *



HP-41CV PROGRAM LISTING (cont.)

93 ST+ 00	139 *
94 CLX	140 -
95 RCL 09	141 STO 20
96 *	142 STO 16
97 RCL 07	143 RCL 25
98 +	144 RCL 07
99 RCL 06	145 *
100 +	146 RCL 24
101 *	147 RCL 05
102 ST+ 04	148 *
103 CLX	149 -
104 RCL 03	150 STO 21
105 *	151 STO 17
106 RCL 08	152 RCL 25
107 ENTER↑	153 RCL 08
108 +	154 *
109 +	155 RCL 24
110 *	156 RCL 06
111 ST+ 01	157 *
112 CLX	158 -
113 RCL 02	159 STO 22
114 *	160 STO 18
115 ST+ 05	161 RCL 25
116 CLX	162 RCL 20
117 RCL 09	163 *
118 *	164 RCL 24
119 ST+ 06	165 RCL 19
120 ST+ 07	166 *
121 CLX	167 -
122 RCL 03	168 RCL 10
123 *	169 RCL 28
124 ST+ 08	170 -
125 RCL 25	171 X↑2
126 RCL 04	172 RCL 11
127 *	173 RCL 29
128 RCL 24	174 -
129 RCL 00	175 X↑2
130 *	176 +
131 -	177 RCL 23
132 STO 19	178 *
133 STO 15	179 +
134 RCL 25	180 ST/ 15
135 RCL 01	181 ST/ 16
136 *	182 ST/ 17
137 RCL 24	183 ST/ 18
138 RCL 04	184 RCL 29

HP-41CV PROGRAM LISTING (cont.)

185 RCL 11	231 RCL 15
186 -	232 RCL 21
187 RCL 25	233 *
188 *	234 ST- 05
189 RCL 28	235 RCL 15
190 RCL 10	236 RCL 22
191 -	237 *
192 RCL 24	238 ST- 06
193 *	239 RCL 16
194 -	240 RCL 21
195 STO 33	241 *
196 RCL 15	242 ST- 07
197 *	243 RCL 16
198 ST+ 10	244 RCL 22
199 RCL 33	245 *
200 RCL 16	246 ST- 08
201 *	247 RCL 17
202 ST+ 11	248 RCL 22
203 RCL 33	249 *
204 RCL 17	250 ST- 09
205 *	251 GTO "T8"
206 ST+ 12	252+LBL "LEG"
207 RCL 33	253 "LEG BRG?"
208 RCL 18	254 PROMPT
209 *	255 "LEG DST?"
210 ST+ 13	256 PROMPT
211 RCL 15	257 RCL 39
212 RCL 19	258 /
213 *	259 P-R
214 ST- 00	260 ST+ 28
215 RCL 16	261 X<>Y
216 RCL 20	262 ST+ 29
217 *	263 SF 09
218 ST- 01	264 GTO "CS"
219 RCL 17	265+LBL "FIX"
220 RCL 21	266 SF 21
221 *	267 FIX 1
222 ST- 02	268 RCL 13
223 RCL 16	269 RCL 12
224 RCL 22	270 R-P
225 *	271 STO 34
226 ST- 03	272 XEQ 91
227 RCL 15	273 "TGT CUS: "
228 RCL 20	274 ARCL X
229 *	275 AVIEW
230 ST- 04	276 "TGT SPD: "

HP-41CV PROGRAM LISTING (cont.)

277 ARCL 34	312 RCL 04
278 AVIEW	313 X+2
279 RCL 11	314 +
280 RCL 29	315 SORT
281 -	316 STO 34
282 RCL 10	317 +
283 RCL 28	318 SORT
284 -	319 "SIG1: "
285 R-P	320 ARCL X
286 RCL 39	321 AVIEW
287 *	322 RCL 33
288 STO 34	323 RCL 34
289 XEQ 91	324 -
290 "TGT BRG: "	325 SORT
291 ARCL X	326 "SIG2: "
292 AVIEW	327 ARCL X
293 "TGT RNG: "	328 AVIEW
294 ARCL 34	329 RCL 04
295 AVIEW	330 ST+ X
296 GTO 99	331 RCL 00
297*LBL "AOP"	332 RCL 01
298 SF 21	333 -
299 FIX 2	334 R-P
300 RCL 00	335 XEQ 91
301 RCL 01	336 2
302 +	337 /
303 2	338 "ANG: "
304 /	339 ARCL X
305 STO 33	340 AVIEW
306 RCL 00	341 GTO 99
307 RCL 01	342*LBL 91
308 -	343 CLX
309 2	344 360
310 /	345 MOD
311 X+2	346 END

### III. THE HEWLETT-PACKARD HP-75C.

#### HP-75C USER INSTRUCTIONS.

There are four user options. These options - labeled 1, 2, 3, and 4 - are used to continue input, input a new leg, compute a fix, and to compute an AOP, respectively. Their use is shown below. (Memory: 2727 bytes for program; 820 bytes for variables. Total = 3547 bytes.)

DISPLAY	INSTRUCTION	INPUT
1.	<b>Run Program.</b>	Run "TMA"
2. Units:	Select units:	
1 n.mi.,	Nautical miles,	1
2 yds., or	yards, or	2
3 m.?	meters.	3
3. BRG ERR?	Key in bearing error.	[RTN]
4. OWN CUS?	Key in own course.	[RTN]
5. OWN SPD?	Key in own speed.	[RTN]
6. TGT BRG?	Key in target bearing.	[RTN]
7. TIME?	Key in time (HH.MM).	[RTN]
8. 1 CONT,	1 returns to Step 6.	1
2 LEG,	2 goes to Step 9.	2
3 FIX,	3 goes to Step 13.	3
4 AOP?	4 goes to Step 18.	4
9. LEG BRG?	Key in new leg bearing.	[RTN]
10. LEG DST?	Key in distance on leg.	[RTN]
12.	Go to Step 4.	
13.	TGT CUS: (degrees)	[RTN]
14.	TGT SPD: (knots)	[RTN]
15.	TGT BRG: (degrees)	[RTN]
16.	TGT RNG: (units)	[RTN]
17.	Go to Step 8.	
18.	SIG 1: (units)	[RTN]
19.	SIG 2: (units)	[RTN]
20.	ANG: (degrees)	[RTN]
21.	Go to Step 8.	

HP-75C SAMPLE PROBLEM

The sensor on your ship has a bearing error of one degree. Your course is 160 deg. at 6 kts. At 1200 hours your sensor detects a target at 350.5 degrees. At 1204 the sensor indicates the target to be at 1.8 deg. To determine a fix, your ship makes a course change during which the course-made-good is 130 deg. and the distance-made-good is 556 meters. Your new course is 080 deg. at 6 kts. The maneuver ends at 1207, at which time the target bearing is 8.3 deg. One more observation places the target at 18.4 deg. at 1211. Estimate the target course, speed, bearing, range, and AOP at 1211 hours.

DISPLAY CONTENTS	USER RESPONSE	COMMENTS
Units: 1 n.mi., 2 yds., or 3 m.?	3	Select meters.
BRG ERR?	1 [RTN]	1 deg.
OWN CUS?	160 [RTN]	160 deg.
OWN SPD?	6 [RTN]	6 kts.
TGT BRG?	350.5 [RTN]	350.5 deg.
TIME?	12.00 [RTN]	1200 hours.
1 CONT, 2 LEG, 3 FIX, 4 AOP?	1	Continue input.
TGT BRG?	1.8 [RTN]	1.8 deg.
TIME?	12.04 [RTN]	1204 hours.
1 CONT, 2 LEG, 3 FIX, 4 AOP?	2	Input new leg.
LEG BRG?	130 [RTN]	130 deg.
LEG DST?	556 [RTN]	556 meters.
OWN CUS?	080 [RTN]	080 deg.
OWN SPD?	6 [RTN]	6 kts.
TGT BRG?	8.3 [RTN]	8.3 deg.
TIME?	12.07 [RTN]	1207 hours.
1 CONT, 2 LEG, 3 FIX, 4 AOP?	1	Continue input.
TGT BRG?	18.4 [RTN]	18.4 deg.
TIME?	12.11 [RTN]	1211 hours.
1 CONT, 2 LEG, 3 FIX, 4 AOP?	3	Compute a fix.
CUS = 123.7		course = 123.7 deg.
SPD = 12.6		speed = 12.6 kts.
BRG = 18.4		bearing = 18.4 deg.
RNG = 4023 m.		range = 4023 meters.
1 CONT, 2 LEG, 3 FIX, 4 AOP?	4	Compute an AOP.
$\sigma$ 1 = 19263.71 m.		sigma 1 = 19263.71 m.
$\sigma$ 2 = 105.22 m.		sigma 2 = 105.22 m.
ANG = 18.4		angle = 18.4 deg.
1 CONT, 2 LEG, 3 FIX, 4 AOP?		Quit.

# HP-75C PROGRAM LISTING

```

10 REM TMA
20 GOTO 60
30 C#=KEY# @ IF C#=' ' TH
EN 30
40 FOR C=1 TO LEN(C0#) @
IF C#=C0#[C,C] THEN RET
URN
50 NEXT C @ GOTO 30
60 OPTION BASE 1 @ OPTIO
N ANGLE DEGREES
70 DIM P(10),H(4),X(4)
80 DEF FNR(X,M) = INT(M*
X+.5)/M
90 X0=0 @ Y0=0 @ F5=0
100 F0=1 @ T9=0 @ L=0 @
FOR I=1 TO 4 @ FOR J=I T
O 4 @ L=L+1 @ T=0 @ IF I
=J THEN T=1000
110 P(L)=T @ NEXT J @ NE
XT I
120 DISP 'Units: 1 n.mi,
2 yds, or 3 n.? ' @ C0#
='123' @ GOSUB 30 @ U=C
130 IF U=1 THEN K=1 @ U#
='n' @ GOTO 160
140 IF U=2 THEN K=2025 @
U#='yds.' @ GOTO 160
150 IF U=3 THEN K=1852 @
U#='n.' @ GOTO 160
160 GOSUB 4000 @ GOSUB 4
010 @ GOSUB 4020 @ GOSUB
4030 @ GOSUB 4040
170 GOTO 240
180 DISP '1 CONT, 2 LEG,
3 FIX, 4 AOP?' @ C0#='1
234' @ GOSUB 30
190 IF C=2 THEN GOSUB 40
70 @ GOTO 220
200 IF C=3 THEN GOSUB 10
00 @ GOTO 180
210 IF C=4 THEN GOSUB 10
50 @ GOTO 180
220 IF F5=1 THEN GOSUB 4
010
230 GOSUB 4020 @ GOSUB 4
040
240 X(1)=X(1)+X(3)*D9 @
X(2)=X(2)+X(4)*D9
250 P(1)=P(1)+(2*P(3)+P(
8)*D9)*D9
260 P(2)=P(2)+(P(4)+P(6)
+P(9)*D9)*D9
270 P(5)=P(5)+(2*P(7)+P(
10)*D9)*D9
280 P(3)=P(3)+P(8)*D9
290 P(4)=P(4)+P(9)*D9
300 P(6)=P(6)+P(9)*D9
310 P(7)=P(7)+P(10)*D9
320 H(1)=C*P(2)-S*P(1)
330 H(2)=C*P(5)-S*P(2)
340 H(3)=C*P(6)-S*P(3)
350 H(4)=C*P(7)-S*P(4)
360 D=C*H(2)-S*H(1)+E*((
X(1)-X0)^2+(X(2)-Y0)^2)
370 Z=Y0*C-X0*S @ S5=Z-(
C*X(2)-S*X(1))
380 FOR I=1 TO 4 @ X(I)=
X(I)+H(I)/D*S5 @ NEXT I
390 L=0 @ FOR I=1 TO 4 @
FOR J=I TO 4 @ L=L+1 @
P(L)=P(L)-H(I)*H(J)/D @
NEXT J @ NEXT I
400 GOTO 180
1000 X2=X(1)-X0 @ Y2=X(2
)-Y0 @ R2=SQR(X2*X2+Y2*Y
2) @ B2=ANGLE(X2,Y2) @ I
F B2<0 THEN B2=B2+360
1010 X2=X(3) @ Y2=X(4) @
S2=SQR(X2*X2+Y2*Y2) @ C
2=ANGLE(X2,Y2) @ IF C2<0
THEN C2=C2+360
1020 DISP 'CUS =';FNR(C2
,10); @ DISP ' SPD =';FN
R(S2,10) @ GOSUB 4100
1030 DISP 'BRG =';FNR(B2
,10); @ DISP ' RNG =';FN
R(R2*K,10);U# @ GOSUB 41
00
1040 RETURN
1050 A=P(1) @ B=P(5) @ D
=P(2) @ A1=ANGLE(A-B,2*D
)/2 @ IF A1<0 THEN A1=A1
+180
1060 S3=(A+B)/2 @ C3=SQR
((A-B)^2/4+D*D) @ S1=S3+
C3

```

HP-75C PROGRAM LISTING (cont.)

```

1070 S2=S3-C3
1080 DISP ' 1 = ';FNR(K
*SQR(S1),100);U# @ GOSUB
 4100
1090 DISP ' 2 = ';FNR
(K*SQR(S2),100);U# @ GOS
UB 4100
1100 DISP 'ANG = ';FNR(A
1,10) @ GOSUB 4100 @ RET
URN
4000 INPUT 'BRG ERR? ';
E @ E=RAD(E) @ E=E*E @ R
ETURN
4010 INPUT 'OWN CUS? ';
C0 @ INPUT 'OWN SPD? ';
S0 @ RETURN
4020 INPUT 'TGT BRG? ';
B9 @ S=SIN(B9) @ C=COS(B
9) @ RETURN
4030 R9=32 @ X(1)=R9*C @
X(2)=R9*S @ X(3)=0 @ X(
4)=0 @ RETURN @ REM R9 =
EST RNG
4040 T0=T9 @ INPUT 'TIME
? '; T9 @ T9=FNR(T9) @ I
F F0=1 THEN F0=0 @ T0=T9
4050 D9=T9-T0 @ IF F5=0
THEN X0=X0+D9*S0*COS(C0)
@ Y0=Y0+D9*S0*SIN(C0)
4060 F5=0 @ RETURN
4070 INPUT 'LEG BRG? ';
B8 @ B8=FNR(B8) @ INPUT
'LEG DST? '; R8 @ R8=R8/
K
4080 X0=X0+R8*COS(B8) @
Y0=Y0+R8*SIN(B8)
4090 F5=1 @ RETURN
4100 WAIT 5 @ RETURN
4110 DEF FNR(V)
4120 H=INT(V) @ V=FP(V)*
100 @ FNR=(100*FP(V)/60+
INT(V))/60+H
4130 END DEF

```

IV. THE SHARP PC-1500 (RADIO SHACK TRS-80 PC-2).

SHARP PC-1500 (TRS-80 PC-2) USER INSTRUCTIONS.

There are three main user options. These options - labeled 1, 2, and 3 - are used to continue input, input a new leg, and to compute a fix, respectively. Other options include unit selection (n. mi., yds. or meters) and AOP selection. Their use is shown below. (Memory: 2050 bytes for program; number of bytes for variables is not available.)

DISPLAY	INSTRUCTION	INPUT
1.	Run Program.	RUN [ENTER]
2.	Units: 1 n.mi., 2 yds., or 3 meter?	1 2 3
3.	BRG ERR?	[ENTER]
4.	OWN CUS?	[ENTER]
5.	OWN SPD?	[ENTER]
6.	TGT BRG?	[ENTER]
7.	TIME?	[ENTER]
8.	1 CONT, 2 LEG, 3 FIX?	1 2 3
9.	LEG BRG?	[ENTER]
10.	LEG DST?	[ENTER]
12.	Go to Step 4.	
13.	TGT CUS: (degrees)	[ENTER]
14.	TGT SPD: (knots)	[ENTER]
15.	TGT BRG: (degrees)	[ENTER]
16.	TGT RNG: (units)	[ENTER]
17.	AOP? Y or N	Y N
18.	SIG 1: (units)	[ENTER]
19.	SIG 2: (units)	[ENTER]
20.	ANG: (degrees)	[ENTER]
21.	Go to Step 8.	



SHARP PC-1500 (TRS-80 PC-2) SAMPLE PROBLEM

The sensor on your ship has a bearing error of one degree. Your course is 160 deg. at 6 kts. At 1200 hours your sensor detects a target at 350.5 degrees. At 1204 the sensor indicates the target to be at 1.8 deg. To determine a fix, your ship makes a course change during which the course-made-good is 130 deg. and the distance-made-good is 556 meters. Your new course is 080 deg. at 6 kts. The maneuver ends at 1207, at which time the target bearing is 8.3 deg. One more observation places the target at 18.4 deg. at 1211. Estimate the target course, speed, bearing, range, and AOP at 1211 hours.

DISPLAY CONTENTS	USER RESPONSE	COMMENTS
Units: 1 n.mi., 2 yds., or 3 m.?	3	Select meters.
BRG ERR?	1 [ENTER]	1 deg.
OWN CUS?	160 [ENTER]	160 deg.
OWN SPD?	6 [ENTER]	6 kts.
TGT BRG?	350.5 [ENTER]	350.5 deg.
TIME?	12.00 [ENTER]	1200 hours.
1 CONT, 2 LEG, 3 FIX?	1	Continue input.
TGT BRG?	1.8 [ENTER]	1.8 deg.
TIME?	12.04 [ENTER]	1204 hours.
1 CONT, 2 LEG, 3 FIX?	2	Input new leg.
LEG BRG?	130 [ENTER]	130 deg.
LEG DST meters?	556 [ENTER]	556 meters.
OWN CUS?	080 [ENTER]	080 deg.
OWN SPD?	6 [ENTER]	6 kts.
TGT BRG?	8.3 [ENTER]	8.3 deg.
TIME?	12.07 [ENTER]	1207 hours.
1 CONT, 2 LEG, 3 FIX?	1	Continue input.
TGT BRG?	18.4 [ENTER]	18.4 deg.
TIME?	12.11 [ENTER]	1211 hours.
1 CONT, 2 LEG, 3 FIX?	3	Compute a fix.
CUS = 123.7		course = 123.7 deg.
SPD = 12.6		speed = 12.6 kts.
BRG = 18.4		bearing = 18.4 deg.
RNG = 4023 m.		range = 4023 meters.
AOP? Y or N	Y	Compute an AOP.
SIG1 = 19263.71 m.		sigma 1 = 19263.71 m.
SIG2 = 105.22 m.		sigma 2 = 105.22 m.
ANG = 18.39		angle = 18.39 deg.
1 CONT, 2 LEG, 3 FIX?		Quit.

SHARP PC-1500 (TRS-80 PC-2) PROGRAM LISTING

```

10:REM TMA
15:WAIT :GOTO 30
20:C$=INKEY$ :IF
  C$=""THEN 20
22:FOR C=1TO LEN
  (C0$):IF C$=
  MID$ (C0$,C,1)
  THEN RETURN
24:NEXT C:GOTO 20
30:DIM P(10),H(4)
  ,X(4):RD=PI /1
  80:DEGREE
50:X0=0:Y0=0:F5=0
60:F0=1:T9=0:L=0:
  FOR I=1TO 4:
  FOR J=1TO 4:L=
  L+1:T=0:IF I=J
  THEN LET T=100
  0
70:P(L)=T:NEXT J:
  NEXT I
80:PAUSE "1 n.mi,
  2 yds, or 3 m
  eter?"
85:C0$="123":
  GOSUB 20:K=C
90:IF K=1THEN LET
  U$=" n.mi.":
  GOTO 130
100:IF K=2THEN LET
  K=2025:U$=" yd
  s.":GOTO 130
110:IF K=3THEN LET
  K=1852:U$=" me
  ters"
130:GOSUB 4000:
  GOSUB 4010:
  GOSUB 4020:
  GOSUB 4030:
  GOSUB 4040
140:GOTO 210
150:PAUSE "1 CONT,
  2 LEG, 3 FIX?
  ":C0$="123":
  GOSUB 20:CLS :
  WAIT
160:IF C=2THEN
  GOSUB 4070:
  GOTO 190
170:IF C=3THEN
  GOSUB 1000:
  GOTO 150
190:IF F5=1THEN
  GOSUB 4010
200:GOSUB 4020:
  GOSUB 4040
210:X(1)=X(1)+X(3)
  *D9:X(2)=X(2)+
  X(4)*D9
220:P(1)=P(1)+(2*P
  (3)+P(8)*D9)*D
  9
230:P(2)=P(2)+(P(4
  )+P(6)+P(9)*D9
  )*D9
240:P(5)=P(5)+(2*P
  (7)+P(10)*D9)*
  D9
250:P(3)=P(3)+P(8)
  *D9
260:P(4)=P(4)+P(9)
  *D9
270:P(6)=P(6)+P(9)
  *D9
280:P(7)=P(7)+P(10
  )*D9
290:H(1)=C*P(2)-S*
  P(1)
300:H(2)=C*P(5)-S*
  P(2)
310:H(3)=C*P(6)-S*
  P(3)
320:H(4)=C*P(7)-S*
  P(4)
330:D=C*H(2)-S*H(1
  )+E*((X(1)-X0)
  ^2+(X(2)-Y0)^2
  )
340:Z=Y0*C-X0*S:S5
  =Z-(C*X(2)-S*X
  (1))
350:FOR I=1TO 4:X(
  I)=X(I)+H(I)/D
  *S5:NEXT I
360:L=0:FOR I=1TO
  4:FOR J=1TO 4:
  L=L+1:P(L)=P(L
  )-H(I)*H(J)/D:
  NEXT J:NEXT I
370:GOTO 150
1000:X=X(1)-X0:Y=
  X(2)-Y0:R2=
  SQR (X*X+Y*Y
  ):GOSUB 5000
  :B2=AT

```

```

1010: X=X(3): Y=X(4
      ): S2=SQR (X*
      X+Y*Y): GOSUB
      5000: C2=AT
1020: M=10: X=C2:
      GOSUB 5010:
      PRINT "TGT C
      US ="; X: X=S2
      : GOSUB 5010:
      PRINT "TGT S
      PD ="; X
1030: X=B2: GOSUB 5
      010: PRINT "T
      GT BRG ="; X:
      X=R2*K: GOSUB
      5010: PRINT "
      TGT RNG ="; X
      ; U$
1040: PAUSE "AOP?
      Y or N ": C0$
      ="YyNn":
      GOSUB 20: CLS
1050: IF C>2 THEN
      RETURN
1080: A=P(1): B=P(5
      ): D=P(2): X=A
      -B: Y=2*D:
      GOSUB 5000: A
      1=AT/2
1090: S3=(A+B)/2: C
      3=SQR ((A-B)
      ^2/4+D*D)
1100: S1=S3+C3: S2=
      S3-C3
1110: M=100: X=K*
      SQR (S1):
      GOSUB 5010:
      PRINT "SIG1=
      "; X; U$
1120: X=K*SQR (S2)
      : GOSUB 5010:
      PRINT "SIG2
      =" ; X; U$
1130: X=A1: GOSUB 5
      010: PRINT "A
      NG ="; X:
      RETURN
4000: INPUT "BRG E
      RR? "; E: E=E*
      RD: E=E*E:
      RETURN
4010: INPUT "OWN C
      US? "; C0:
      INPUT "OWN S
      PD? "; S0:
      RETURN
4020: INPUT "TGT B
      RG? "; B9: S=
      SIN B9: C=COS
      B9: RETURN
4030: R9=32: X(1)=R
      9*C: X(2)=R9*
      S: X(3)=0: X(4
      )=0: RETURN
4040: T8=T9: INPUT
      "TIME? "; T9:
      CLS : T9=DEG
      T9: IF F0=1
      THEN LET F0=
      0: T8=T9
4050: D9=T9-T8: IF
      F5=0 THEN LET
      X0=X0+D9*S0*
      COS C0: Y0=Y0
      +D9*S0*SIN C
      0
4060: F5=0: RETURN
4070: INPUT "LEG B
      RG? "; B8: B8=
      DEG B8
4075: WAIT 0: PRINT
      "LEG DST "; U
      $; : INPUT "?
      "; R8: PAUSE :
      R8=R8/K
4080: X0=X0+R8*COS
      B8: Y0=Y0+R8*
      SIN B8
4090: F5=1: RETURN
5000: AT=ATN (Y/(X
      +1E-9*(0=X))
      )+180*(X<0):
      AT=AT+360*(A
      T<0): RETURN
5010: X=INT (M*X+.
      5)/M: RETURN

```

V. THE RADIO SHACK TRS-80 MODEL 100.

RADIO SHACK TRS-80 MODEL 100 USER INSTRUCTIONS.

There are four user options. These options - labeled 1, 2, 3, and 4 - are used to continue input, input a new leg, compute a fix, and to compute an AOP, respectively. Their use is shown below. (Memory: 2503 bytes for program; 815 bytes for variables. Total = 3318 bytes.)

DISPLAY	INSTRUCTION	INPUT
1.	<b>Run Program.</b>	Run [ENTER]
2.	Units: 1 n.mi., 2 yds., or 3 m.?	Select units: Nautical miles, yards, or meters? 1 2 3
3.	BRG ERR?	Key in bearing error. [ENTER]
4.	OWN CUS?	Key in own course. [ENTER]
5.	OWN SPD?	Key in own speed. [ENTER]
6.	TGT BRG?	Key in target bearing. [ENTER]
7.	TIME?	Key in time (HH.MM). [ENTER]
8.	1 Continue, 2 Leg, 3 Fix, or 4 AOP?	1 returns to Step 6. 2 goes to Step 9. 3 goes to Step 13. 4 goes to Step 18. 1 2 3 4
9.	LEG BRG?	Key in new leg bearing. [ENTER]
10.	LEG DST?	Key in distance on leg. [ENTER]
12.		Go to Step 4.
13.		TGT CUS: (degrees) [ENTER]
14.		TGT SPD: (knots) [ENTER]
15.		TGT BRG: (degrees) [ENTER]
16.		TGT RNG: (units) [ENTER]
17.		Press any key to continue. [ENTER] Go to Step 8.
18.		SIG 1: (units) [ENTER]
19.		SIG 2: (units) [ENTER]
20.		ANG: (degrees) [ENTER]
21.		Press any key to continue. [ENTER] Go to Step 8.

RADIO SHACK TRS-80 MODEL 100 SAMPLE PROBLEM

The sensor on your ship has a bearing error of one degree. Your course is 160 deg. at 6 kts. At 1200 hours your sensor detects a target at 350.5 degrees. At 1204 the sensor indicates the target to be at 1.8 deg. To determine a fix, your ship makes a course change during which the course-made-good is 130 deg. and the distance-made-good is 556 meters. Your new course is 080 deg. at 6 kts. The maneuver ends at 1207, at which time the target bearing is 8.3 deg. One more observation places the target at 18.4 deg. at 1211. Estimate the target course, speed, bearing, range, and AOP at 1211 hours.

DISPLAY CONTENTS	USER RESPONSE	COMMENTS
Units: 1 n.mi., 2 yds., or 3 m.?	3	Select meters.
BRG ERR?	1 [ENTER]	1 deg.
OWN CUS?	160 [ENTER]	160 deg.
OWN SPD?	6 [ENTER]	6 kts.
TGT BRG?	350.5 [ENTER]	350.5 deg.
TIME?	12.00 [ENTER]	1200 hours.
1 Continue, 2 Leg, 3 Fix, or 4 AOP?	1	Continue input.
TGT BRG?	1.8 [ENTER]	1.8 deg.
TIME?	12.04 [ENTER]	1204 hours.
1 Continue, 2 Leg, 3 Fix, or 4 AOP?	2	Input new leg.
LEG BRG?	130 [ENTER]	130 deg.
LEG DST meters?	556 [ENTER]	556 meters.
OWN CUS?	080 [ENTER]	080 deg.
OWN SPD?	6 [ENTER]	6 kts.
TGT BRG?	8.3 [ENTER]	8.3 deg.
TIME?	12.07 [ENTER]	1207 hours.
1 Continue, 2 Leg, 3 Fix, or 4 AOP?	1	Continue input.
TGT BRG?	18.4 [ENTER]	18.4 deg.
TIME?	12.11 [ENTER]	1211 hours.
1 Continue, 2 Leg, 3 Fix, or 4 AOP?	3	Compute a fix.
CUS = 123.7		course = 123.7 deg.
SPD = 12.6		speed = 12.6 kts.
BRG = 18.4		bearing = 18.4 deg.
RNG = 4023 m.		range = 4023 meters.
1 Continue, 2 LEG, 3 Fix, or 4 AOP?	4	Compute an AOP.
SIG 1 = 19263.71 m.		sigma 1 = 19263.71 m.
SIG 2 = 105.22 m.		sigma 2 = 105.22 m.
ANG = 18.39		angle = 18.39 deg.
1 Continue, 2 LEG, 3 Fix, or 4 AOP?		Quit.

TRS-80 MODEL 100 PROGRAM LISTING

```

10 REM TMA
20 DIMP(4,4),H(4),X(4):PI=4*ATN(1):TP=PI+PI:RD=PI/180:GOTO50
25 FORC=1TO9:C$=INKEY$:NEXTC
26 PRINT:PRINT"Press any key to continue."
27 C$=INKEY$:IFC$=""GOTO27
28 RETURN
32 FORC=1TO9:C$=INKEY$:NEXTC
33 C$=INKEY$:IFC$=""GOTO33
34 C=ASC(C$):C$=CHR$(C+32*(C>90))
35 FORC=1TOLEN(C0$):IFC$=MID$(C0$,C,1)THENRETURN
36 NEXTC:GOTO33
40 X=INT(M*X+.5)/M:RETURN:REM ROUND
50 X0=0:Y0=0:F5=0
60 F0=1:T9=0:FORI=1TO4:FORJ=1TO4:T=0:IFI=JTHEN T=1000
70 P(I,J)=T:NEXTJ:NEXTI
80 CLS:PRINT"Units:":PRINT" 1 - n.mi.,"
82 PRINT" 2 - yds, or":PRINT" 3 - meters?"
85 C0$="123":GOSUB32:K=C
90 IFK=1THEN U$=" n.mi.":GOTO130
100 IFK=2THEN K=2025:U$=" yds.":GOTO130
110 IFK=3THEN K=1852:U$=" meters."
130 GOSUB4000:GOSUB4010:GOSUB4020:GOSUB4030:GOSUB4040
140 GOTO210
150 CLS:PRINT"Select Option:":PRINT" 1 - Continue,"
152 PRINT" 2 - New Leg,":PRINT" 3 - Fix, or":PRINT" 4 - AOP?"
153 C0$="1234":GOSUB32:CLS
160 IFC=2THEN GOSUB4070:GOTO190
170 IFC=3THEN GOSUB1000:GOTO150
180 IFC=4THEN GOSUB1080:GOTO150
190 IFF5=1THEN GOSUB4010
200 GOSUB4020:GOSUB4040
210 X(1)=X(1)+X(3)*D9:X(2)=X(2)+X(4)*D9
220 P(1,1)=P(1,1)+(2*P(1,3)+P(3,3)*D9)*D9
230 P(1,2)=P(1,2)+(P(1,4)+P(2,3)+P(3,4)*D9)*D9
240 P(2,2)=P(2,2)+(2*P(2,4)+P(4,4)*D9)*D9
250 P(1,3)=P(1,3)+P(3,3)*D9
260 P(1,4)=P(1,4)+P(3,4)*D9
270 P(2,3)=P(2,3)+P(3,4)*D9
280 P(2,4)=P(2,4)+P(4,4)*D9
290 FORI=1TO4:FORJ=1TO4:P(J,I)=P(I,J):NEXTJ:NEXTI

```

```

300 H(1)=C*P(1,2)-S*P(1,1)
310 H(2)=C*P(2,2)-S*P(2,1)
320 H(3)=C*P(3,2)-S*P(3,1)
330 H(4)=C*P(4,2)-S*P(4,1)
340 D=C*H(2)-S*H(1)+E*((X(1)-X0)^2+(X(2)-Y0)^2)
350 Z=Y0*C-X0*S:S5=Z-(C*X(2)-S*X(1))
360 FORI=1TO4:X(I)=X(I)+H(I)/D*S5:NEXTI
370 FORI=1TO4:FORJ=1TO4:P(I,J)=P(I,J)-H(I)*H(J)/D:NEXTJ:NEXTI
380 GOTO150
1000 X=X(1)-X0:Y=X(2)-Y0:R2=SQR(X*X+Y*Y):GOSUB5000:B2=AT
1010 X=X(3):Y=X(4):S2=SQR(X*X+Y*Y):GOSUB5000:C2=AT
1020 M=10:X=C2/RD:GOSUB40:PRINT" TGT CUS
=";X:X=S2:GOSUB40:PRINT" TGT SPD =" ;X
1030 X=B2/RD:GOSUB40:PRINT" TGT BRG =" ;X:X=R2*K:GOSUB40:PRINT" TGT
RNG =" ;X;U$
1040 GOSUB25:RETURN
1080 A=P(1,1):B=P(2,2):D=P(1,2):X=A-B:Y=D+D:GOSUB5000:A1=AT/2
1090 S3=(A+B)/2:C3=SQR((A-B)^2/4+D*D):S1=S3+C3
1100 S2=S3-C3
1110 M=100:X=K*SQR(S1):GOSUB40:PRINT" SIG 1 =" ;X;U$
1120 X=K*SQR(S2):GOSUB40:PRINT" SIG 2 =" ;X;U$
1130 X=A1/RD:GOSUB40:PRINT" ANG =" ;X:GOSUB25:RETURN
4000 CLS:INPUT" BRG ERR " ;E:E=E*RD:E=E*E:RETURN
4010 INPUT" OWN CUS " ;C0:C0=C0*RD:INPUT" OWN SPD " ;S0:RETURN
4020 INPUT" TGT BRG " ;B9:B9=B9*RD:S=SIN(B9):C=COS(B9):RETURN
4030 R9=32:X(1)=R9*C:X(2)=R9*S:X(3)=0:X(4)=0:RETURN
4040
T8=T9:INPUT" TIME " ;T9:CLS:X=T9:GOSUB4110:T9=V:IFF0=1THENF0=0:T8=T9

4050 D9=T9-T8:IFF5=0THENX0=X0+D9*S0*COS(C0):Y0=Y0+D9*S0*SIN(C0)
4060 F5=0:RETURN
4070 INPUT" LEG BRG " ;B8:B8=B8*RD:PRINT" LEG DST " ;U$ ;:INPUTR8:R8=R8/
K
4080 X0=X0+R8*COS(B8):Y0=Y0+R8*SIN(B8)
4090 F5=1:RETURN
4110 SS=SGN(X):X=ABS(X):H=INT(X):M0=1:GOSUB4200:V=X*
100:X=V:GOSUB4200:
4120 V=SS*((100*X/60+INT(V))/60+H):RETURN
4200 X=X-M0*INT(X/M0):RETURN:REM MOD FCTN
5000 AT=ATN(Y/(X-1E-9*(X=0)))-PI*(X<0):AT=AT-TP*(AT<0):RETURN

```

## VI. REFERENCES

1. R. N. Forrest, "Programs for a Multiple Leg Target Motion Analysis Procedure", Technical Report NPS55-82-026, October 1982, Naval Postgraduate School, Monterey, CA 93940.
2. A. Gelb, editor, Applied Optimal Estimation, The M.I.T. Press, 1974.



## Appendix I: The Fixing Algorithm

The Kalman Filter TMA model may be developed as follows: A target and an observer move on a plane surface. The observer, at position  $(u_k, w_k)$ , measures the target bearing  $B_k$  at time  $t_k$ , for  $k = 1, \dots, n$ . The rectangular coordinate system used to estimate the target's position is shown in Figure 1.

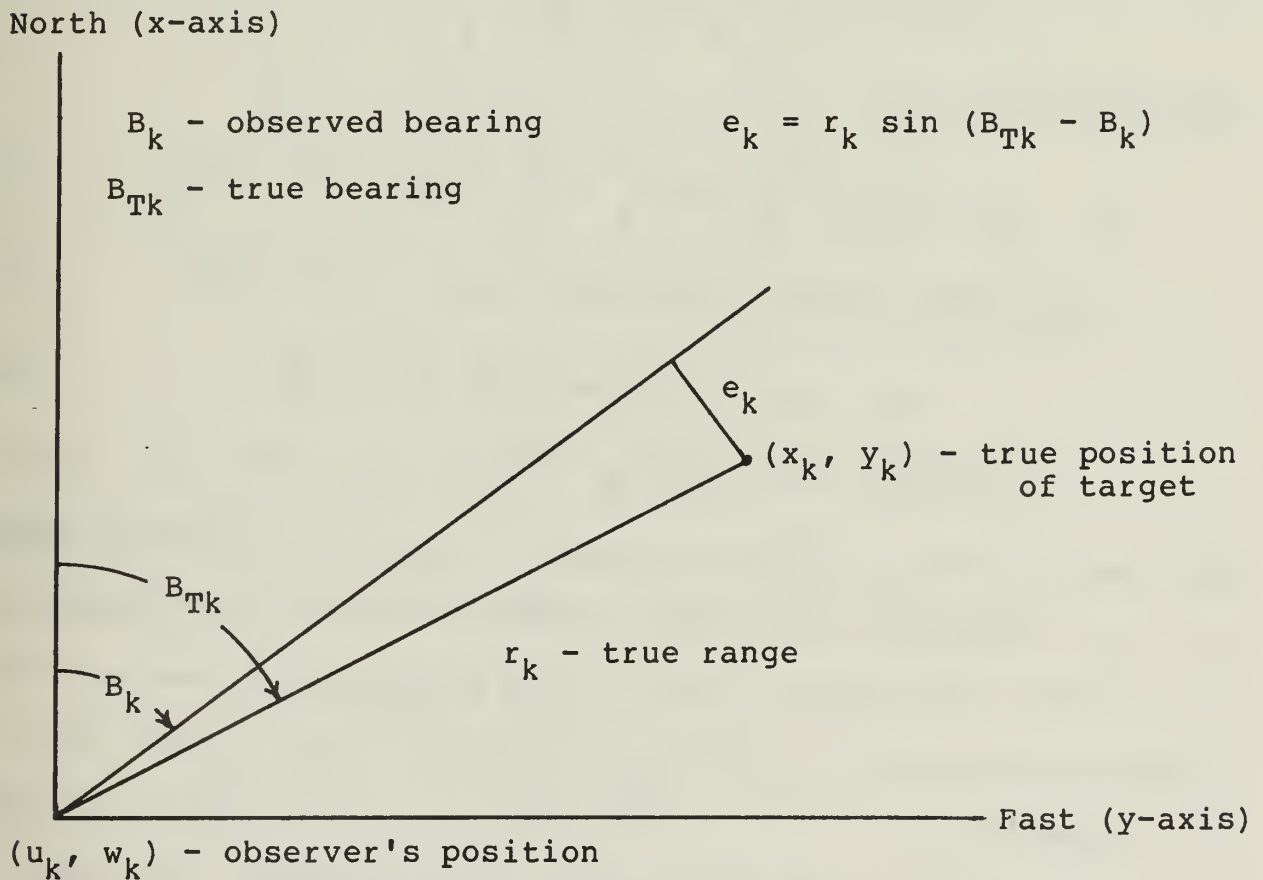


Figure 1. The encounter geometry. The north-south coordinates  $x$  and  $u$  and the east-west coordinates  $y$  and  $w$  are measured from the same origin.

In Figure 1, the point labeled  $(u_k, w_k)$  represents the tracker's position and the point labeled  $(x_k, y_k)$  represents the true target position at  $t_k$ , the time associated with an observed bearing  $B_k$ . At time  $t_k$ ,  $B_{Tk}$  represents the true target bearing and  $r_k$  represents the true target range. The bearing error associated with an observation is assumed to be distributed  $N(0, \sigma_k^2)$ . The error  $e_k$ , at a range  $r_k$ , is then  $N(0, r_k^2 \sigma_k^2)$ .

From Figure 1,

$$e_k = r_k \sin (B_{Tk} - B_k),$$

which rearranges to

$$e_k = u_k \sin B_k - w_k \cos B_k - x_k \sin B_k + y_k \cos B_k. \quad (1)$$

In the least squares procedure [Ref. 1], one sets

$$\begin{aligned} x_k &= x_0 + v_x (t_k - t_0), \\ y_k &= y_0 + v_y (t_k - t_0), \end{aligned} \quad (2)$$

and then minimizes  $\sum e_k^2$  with respect to  $x_0, y_0, v_x$  and  $v_y$ , where  $(x_0, y_0)$  represents the target's position and  $(v_x, v_y)$  represents the target's velocity at time  $t_0$ . The minimizing values are the estimated position and velocity, denoted by  $(\hat{x}_0, \hat{y}_0)$  and  $(\hat{v}_x, \hat{v}_y)$ , respectively.

An identical least squares model is obtained if one sets

$$z_k = x_k \sin B_k - y_k \cos B_k + e_k, \quad (3)$$

where  $z_k$  is computed from

$$z_k = u_k \sin B_k - w_k \cos B_k. \quad (4)$$

Instead of substituting Equ.(2) into Equ.(3) and using the least squares procedure, we treat Equ.(3) as the measurement model in a Kalman Filter [Ref. 2]. The system model is taken to be

$$\mathbf{x}_k = \Phi_{k-1} \mathbf{x}_{k-1}, \quad \text{for } k = 1, 2, \dots$$

where

$$\Phi_k = \begin{bmatrix} 1 & 0 & T_k & 0 \\ 0 & 1 & 0 & T_k \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and

$$\mathbf{x}_k = [x_k \quad y_k \quad v_x \quad v_y]'$$

In these equations,  $T_k = t_{k+1} - t_k$  and  $[...]'$  denotes the transpose matrix.

The measurement model (Equ. 3) can be written as

$$z_k = H_k \mathbf{x}_k + e_k,$$

where  $e_k = N(0, R_k)$ ,  $R_k = r_k^2 \sigma_k^2$ , and

$$H_k = [\sin B_k \quad -\cos B_k \quad 0 \quad 0].$$

If we let ( $\sim$ ) denote an extrapolated quantity (the movement phase of the Kalman Filter) and ( $\hat{\phantom{x}}$ ) denote an updated estimate (following an observation), then, for the extrapolation

$$\begin{aligned}\hat{\mathbf{x}}_k &= \Phi_{k-1} \hat{\mathbf{x}}_{k-1}, \text{ and} \\ \tilde{\mathbf{P}}_k &= \Phi_{k-1} \tilde{\mathbf{P}}_{k-1} \Phi_{k-1}'.\end{aligned}$$

For the measurement update,

$$\begin{aligned}K_k &= \tilde{\mathbf{P}}_k \mathbf{H}_k' [\mathbf{H}_k \tilde{\mathbf{P}}_k \mathbf{H}_k' + R_k]^{-1}, \\ \hat{\mathbf{x}}_k &= \hat{\mathbf{x}}_k + K_k [z_k - \mathbf{H}_k \hat{\mathbf{x}}_k], \text{ and} \\ \hat{\mathbf{P}}_k &= \tilde{\mathbf{P}}_k - K_k (\tilde{\mathbf{P}}_k \mathbf{H}_k')'.\end{aligned}$$

Since the true range  $r_k$  is not known, we approximate  $R_k$  by  $R_k = \hat{r}_k^2 \sigma_k^2$  where

$$\hat{r}_k^2 = (\hat{x}_k - u_k)^2 + (\hat{y}_k - w_k)^2.$$

To initialize the filter, we assume the initial range of the target to be one convergence zone (CZ), about 32 n. mi., with a standard deviation of one CZ, and the initial position is in the direction of the first bearing observation. Further, the initial speed is taken to be zero with a standard deviation of 32 knots. Thus, we set

$$\hat{\mathbf{x}}_1 = [32 \sin B_1 \quad -32 \cos B_1 \quad 0 \quad 0]',$$

and the covariance matrix

$$\tilde{\mathbf{P}}_1 = \begin{bmatrix} 1000 & 0 & 0 & 0 \\ 0 & 1000 & 0 & 0 \\ 0 & 0 & 1000 & 0 \\ 0 & 0 & 0 & 1000 \end{bmatrix}$$

Following the first observation we find  $\hat{x}_1 = \bar{x}_1$ .

In this model the observer's position  $(u_k, w_k)$  at time  $t_k$  is taken to be

$$u_k = u_0 + v_x T_k \cos C_s,$$

and,

$$w_k = w_0 + v_y T_k \sin C_s,$$

where  $(u_0, w_0)$  is the observer's position at time  $t_0$ . It would be easy to modify this algorithm to allow for multiple sensors by generating the sensor's coordinates  $(u_k, w_k)$  at time  $t_k$ .

If the elements of the covariance matrix  $\hat{P}_k$  are denoted by  $(\hat{p}_{ij})$  then the semimajor and semiminor axes,  $\sigma_1$  and  $\sigma_2$ , of the AOP are computed as follows:

$$\sigma_1^2 = (\hat{p}_{11} + \hat{p}_{22})/2 + [(\hat{p}_{11} - \hat{p}_{22})^2/4 + \hat{p}_{12}^2]^{1/2},$$

$$\sigma_2^2 = (\hat{p}_{11} + \hat{p}_{22})/2 - [(\hat{p}_{11} - \hat{p}_{22})^2/4 + \hat{p}_{12}^2]^{1/2},$$

where the orientation angle  $A$  of the semimajor axis of the AOP is given by

$$A = 0.5 \text{ qatan} (2\hat{p}_{12}, \hat{p}_{11} - \hat{p}_{22}).$$

The notation  $\text{qatan}(Y,X)$  denotes  $\arctan(Y/X)$  corrected for quadrant.

## Appendix II: Some Computational Details

Let the elements of the covariance matrices  $\hat{P}$  and  $\tilde{P}$  be denoted by  $(\hat{p}_{ij})$  and  $(\check{p}_{ij})$ , respectively. Then, suppressing the time dependent subscripts, the covariance extrapolation,

$$\tilde{P} = \Phi \hat{P} \Phi'$$

can be expanded into the following components:

$$\begin{aligned} \check{p}_{11} &= \hat{p}_{11} + 2\hat{p}_{13}^T + \hat{p}_{33}^{T^2} \\ \check{p}_{22} &= \hat{p}_{22} + 2\hat{p}_{24}^T + \hat{p}_{44}^{T^2} \\ \check{p}_{12} &= \hat{p}_{12} + (\hat{p}_{14} + \hat{p}_{23})^T + \hat{p}_{34}^{T^2} \\ \check{p}_{13} &= \hat{p}_{13} + \hat{p}_{33}^T \\ \check{p}_{14} &= \hat{p}_{14} + \hat{p}_{34}^T \\ \check{p}_{23} &= \hat{p}_{23} + \hat{p}_{34}^T \\ \check{p}_{24} &= \hat{p}_{24} + \hat{p}_{44}^T \\ \check{p}_{33} &= \hat{p}_{33} \\ \check{p}_{34} &= \hat{p}_{34} \\ \check{p}_{34} &= \hat{p}_{34} \end{aligned}$$

Let the elements of  $\tilde{P}H'$  be denoted by  $(h_i)$ . Then,

$$\begin{aligned} h_1 &= \check{p}_{12} \cos B - \check{p}_{11} \sin B \\ h_2 &= \check{p}_{22} \cos B - \check{p}_{12} \sin B \\ h_3 &= \check{p}_{23} \cos B - \check{p}_{13} \sin B \\ h_4 &= \check{p}_{24} \cos B - \check{p}_{14} \sin B \end{aligned}$$

and,

$$\tilde{H}P'H' = h_2 \cos B - h_1 \sin B.$$

Let the elements of  $K$  be denoted by  $(k_i)$ . Then,

$$k_i = h_i/D \quad \text{for } i = 1, 2, 3, 4$$

where  $D = \tilde{H}P'H' + R$ .

Finally,

$$\hat{p}_{ij} = \check{p}_{ij} - k_i h_j \quad \text{for } i, j = 1, 2, 3, 4.$$

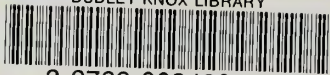
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